PERTURBATIVE COLOR TRANSPARENCY IN ELECTRON BEAM EXPERIMENTS VIA IMPACT-PARAMETER FACTORIZATION

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Color transparency is a prediction of perturbative QCD. Yet detailed calculations have been lacking, and aspects of the required factorization have been controversial. We report on the first complete calculations entirely within a perturbative QCD framework. We also comment on the underlying factorization method and assumptions.

1   Introduction

Color transparency is an important tool to investigate exclusive processes using nuclear targets. The earliest predictions of the phenomenon, going back to 1982,[1] were based on having asymptotically large Q^2 select short distance. Given the difficulties over short-distance dominance, the transition to color transparency then multiplied the controversy. However the discovery of *nuclear filtering* shifted the emphasis.[2,3,4] Components of a hadronic quark wave function with large transverse spatial separations cannot propagate elastically in nuclear matter. This fact of QCD, noted earlier [5] for diffractive processes, was re-discovered for color transparency. It was realized that asymptotically large Q^2 is unnecessary to motivate perturbative color transparency.

Several experiments indicate that color transparency and nuclear filtering have been observed at large nuclear number A. Perhaps the most spectacular is the first color transparency experiment of Carroll *et al*,[6] which convincingly showed that interference effects in proton-proton scattering were filtered away in nuclear targets. The FNAL E-665 experiment [7] also proved consistent with filtering effects,[8] especially in the observation of longitudinal final state polar-

ization in $\gamma^* A \rightarrow \rho A$. Electron beam experiments have been more difficult, partly because of low rates and reduced resolving power.

There are important theoretical reasons not to over-rely on the Q^2 dependence of the transparency ratio.[9] The ratio compares the object of study (an exclusive process in a nuclear target) with something not well-understood (an analogous free-space process). The implicit assumption of a universal hard-scattering form factor with fixed normalization and Q^2 dependence (to be followed by some model of propagation) is theoretically unsupported. Hybrid models, in which free-space form factors are explicitly used, run into the difficulty that exclusive processes are not self-consistently described at laboratory energies by the asymptotic formalism.

Better methods exist to characterize the data empirically. The A dependence is particularly powerful. O'Neill $et\ al$[10] showed that effective attenuation cross sections extracted from $A(e, e'p)$ SLAC data were smaller than Glauber theory calculations by a statistically significant amount. However, choice of normalization and uncertainties in the nuclear spectral distributions complicated the interpretation, and the precision of the data [11] was insufficient to establish a large effect. New $(e, e'p)$ beam experiments are underway at Jefferson Laboratory.

The theory to describe color transparency in pQCD actually forced a revision of basic factorization methods.[3] It was found that the asymptotic factorization of Lepage and Brodsky [12] (LB) was inadequate. To go beyond this, it was necessary to introduce an integration over the transverse separation of quarks into the description. An antecedent was an important paper by Botts and Sterman.[13] The promising new factorization methods, which might be called "impact parameter factorization", have been highly refined in applications by Li and Sterman [14,15] to free-space form factors. Kundu $et\ al$ [16] developed the method further, and responded to criticisms.[17]

The purpose of the transverse integrations is to incorporate regions of finite quark spatial separation. In free-space the Sudakov form factor tends to suppress these regions somewhat; in nuclei a stronger suppression comes from the filtering effects of nuclear matter.[18] The new method also enables calculation of helicity-flip form factors,[19] which cannot be described in the old formalism, and which would be very interesting to measure in nuclear matter, if somehow it could be done.

Happily one does not give up the impulse approximation, which has been the cornerstone of all successful pQCD calculations. Often misconstrued to be the same as the "frozen approximation" of high-energy diffractive processes,

the impulse approximation allows one to start a clock allowing time evolution of the outgoing system, while separating the fast time scale of the scattering from the slow time scale of hadron formation. Controversies in the literature still rage over whether "expansion" in one form or the other has been incorporated properly.[4] Our calculations in perturbation theory integrate over light-cone "minus" components of wavefunctions. This is the step which separates the hadron time scale from the rest. The rest of the calculation is Feynman diagrams, which have the time evolution of quarks built in. Feynman diagrams faithfully reproduce time-evolution of the quark and gluon degrees of freedom order-by-order, creating the same kind of amplitude (a Green function) that the experiment can measure. The physical picture of the impact-parameter factorization, then, is somewhat different from the picture of the asymptotic limit. In the asymptotic limit, quarks participate at zero distance, and move so fast that perfect transparency always occurs. In impact-parameter factorization, quarks with all sideways and longitudinal separations are superposed coherently over the whole nucleus. The hard scattering and the propagation remain coupled during propagation.

2   The Method

It is worthwhile to review the different frameworks of exclusive processes in free space before introducing nuclear targets. Lepage and Brodsky [12] (LB) calculate a meson electromagnetic form factor with a factorization written as

$$F_\pi(Q^2) = \int dx_1 dx_2 \phi(x_2, Q) H(x_1, x_2, Q) \phi(x_1, Q). \tag{1}$$

Here $\phi(x, Q)$ are the distribution amplitudes, which can be expressed in terms of the pion wave function $\psi(x, \vec{k}_T)$ as $\phi(x, Q) = \int^Q d^2 k_T \psi(x, \vec{k}_T)$. We use x for the longitudinal momentum fraction and $\vec{k}_T$ for the transverse momentum carried by the quark. The factorization is justified provided the external photon momentum Q^2 is *asymptotically large*. Then the k_T integrals of perturbation theory decouple, and can be applied to make the distribution amplitudes. The k_T dependence of the hard scattering H can be expanded in a power series, retaining the trivial, constant term. One directly obtains the power-law scaling of the quark-counting method,[20] with logarithmic corrections.

Note that the asymptotic limit is taken right away, in fact prematurely. All distances separating quarks then become asymptotically short. For the purposes of color transparency, taking Q^2 arbitrarily large (but fixed), one might think all targets become perfectly transparent. But then taking $A \to \infty$ at fixed, arbitrarily large Q^2, we know that all targets must become opaque. Thus

there is a limit interchange problem in the LB factorization, because the limit of large Q^2 and large A do not commute. The scheme is fundamentally limited to asymptotic Q^2, and there is no way to fix it to describe the phenomena of color transparency at laboratory energies.

A proper description of the phenomena follows from a impact-parameter factorization scheme incorporating the transverse degrees of freedom[3,13] By including a broader integration region, impact-parameter factorization is more general than the LB method. Li and Sterman [14] simplify the calculation of form-factors by dropping the weak k_T dependence of quark propagators in a hard scattering kernel H. Working in configuration (impact-parameter b) space the expression for a form-factor becomes:

$$F_\pi(Q^2) = \int dx_1 dx_2 \frac{d^2\vec{b}}{(2\pi)^2} \mathcal{P}(x_2, b, P_2, \mu) \tilde{H}(x_1, x_2, Q^2, \vec{b}, \mu) \mathcal{P}(x_1, b, P_1, \mu), \quad (2)$$

where $\mathcal{P}(x, b, P, \mu)$ and $\tilde{H}(x_1, x_2, Q^2, \vec{b}, \mu)$ are the Fourier transforms of the wave function, including Sudakov factors, and hard scattering respectively; $\vec{b}$ is conjugate to $\vec{k}_{T1} - \vec{k}_{T2}$, μ is the renormalization scale and P_1, P_2 are the initial and final momenta of the meson.

We now discuss filtering. When a fast hadron traverse the nuclear medium, each quark interacts primarily through exchange of transverse momentum. While longitudinal momentum can also be exchanged, this degree of freedom does not affect the overall structure, and is in fact lost in the present uncertainties of Feynman x wave-functions. Nuclear targets provide a much better transverse filter than free space, and so it was thought at first that the LB method might apply. The profound differences between impact-parameter factorization and the LB factorization was not understood at first. It was thought that the Ralston and Pire 1990 [3] method was a way to generate distribution amplitudes within the LB framework. Eventually we realized that the large Q^2 dependence of the impact-parameter method is vastly more flexible than just logarithmic behavior, while LB factorization cannot be otherwise.

Transverse momentum integrations conserving overall momentum, and at small momentum transfer, turn into products in b space. Thus the nuclear medium modifies the quark wave function such that [3] $\mathcal{P}_A(x, b, P, \mu) = f_A(b; B)\mathcal{P}(x, b, P, \mu)$, where $\mathcal{P}_A$ is the wave function probed inside the medium and f_A is the nuclear filtering amplitude. An eikonal form [3] appropriate for f_A is: $f_A(b; B) = exp(-\int_z^\infty dz' \sigma(b) \rho(B, z')/2)$. Here $\rho(B, z')$ is the nuclear number density at longitudinal distance z' and impact parameter B relative to the nuclear center. We parametrize $\sigma(b)$ as kb^2 for our calculations. Finally,

we must include the probability to find a target at position B, z inside the nucleus. Putting together the factors, the process of knocking out a hadron from inside a nuclear target has an amplitude M given by

$$M = \int_0^\infty d^2B \int_{-\infty}^{+\infty} dz \rho(B,z) \times F_\pi(x_1, x_2, b, Q^2) \times f_A(b, B) \qquad (3)$$

For the proton the important transverse scale is the maximum of the three quark separation distances, $b_{max} = max(b_1, b_2, b_3)$. The calculation of the process in the nuclear target needs a 9 dimensional integration, which is performed by Monte Carlo. The calculation of proton targets, then, follows the same basic rules as knocking out pions, but with more degrees of freedom. In our calculations we found that uncertainties on the nuclear correlations at the 10% level were a major concern, in some cases exceeding the theoretical uncertainties from the rest of the calculation.[18] We also find that the physics cannot reasonably be captured by a free-space hard scattering, followed by some model of propagation with or without "expansion". This is because the integrations over the transverse quark variable extend over the whole volume of the nucleus. Color transparency is something probing the internal structure of hadrons.

3   Results and Discussions

We calculated color transparency and nuclear filtering for populations of both pions and nucleons (protons) in the nucleus. We explored the Q^2 dependence of the proton transparency ratio using popular distribution amplitude models, and later (after the meeting) with the asymptotic and other models. Realizing belatedly that wave functions more central in Feynman x would possibly be more purely short-distance, we explored such cases and include them for comparison. To make our trial transparency ratio, we divided the rate in the nuclear target by a cross section including the free-space form factor.

Some results are shown in Fig. 1. It is noteworthy that the *slope* of the ratio versus Q^2 depends on the wave-functions. The ratio for a model having a more-central wave function rises faster than that of model with large endpoint contributions. This is consistent with the known tendency for endpoint regions of the CZ and KS models[21] to enhance soft regions of integration, or "big-fat" protons. The asymptotic model, for example, is seen to look like a "smaller" proton in the calculation with the quarks sitting in the central regions.

We are not concerned here with controversies over different trial wave functions, and use the asymptotic one merely to illustrate a point. It is quite
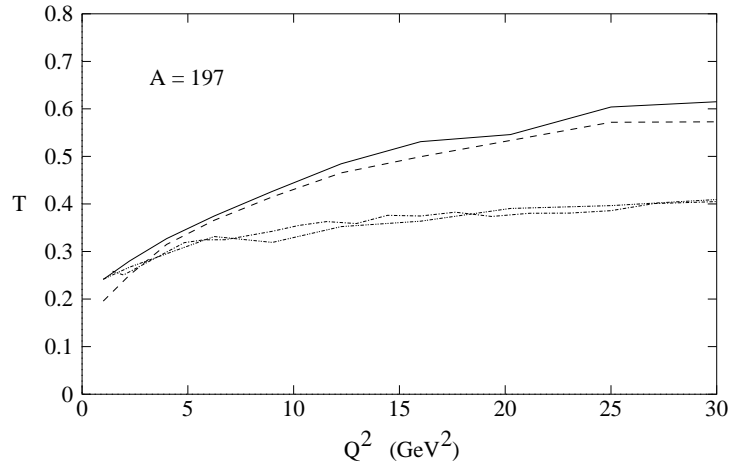
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Figure 1: Color transparency ratio for different proton trial wave functions. Solid curve: asymptotic wave function. Dashed curve: a more centrally peaked wave function of the type $x_1^2 * x_2^2 * x_3^2$. Dash-dot curve: KS wave function. Dash-dot-dot curve: CZ wave function.

striking, and possibly important, that the Q^2 dependence of the transparency ratio measures something as fundamental as the basic x depedence of the wave functions. The calculations indicate that the slope can distinguish centrally peaked from endpoint-peaked models.

For the endpoint-dominated models, the quark transverse separation cut-off showed a significant reduction in sensitivity inside the nucleus compared to free space. However, it must be remembered that the effects of filtering in electron beam experiments are comparatively modest compared to hadron-hadron reactions, because the hadron tends to be knocked out the "back-side" of the nucleus. This accounts for an intrinsic reduced sensitivity of electron beam reactions, which hopefully is compensated by their high intrinsic precision.

Finally, following,[9] we extracted the effective attenuation cross section $\sigma_{eff}(Q^2)$, which serve as a litmus test of whether "color transparency" has actually been achieved. The results (Fig. 2) show a significant decrease of $\sigma_{eff}(Q^2)$ with increasing Q^2 to values well below the Glauber model attenuation cross section. This indicates color transparency. Consistently, the transparency of the asymptotic model is more dramatic than the cases of the CZ and KS models.

Uncertainties in nuclear spectral functions remain, and are becoming problematic for calculation of absolute normalizations. This strongly affects the Q^2
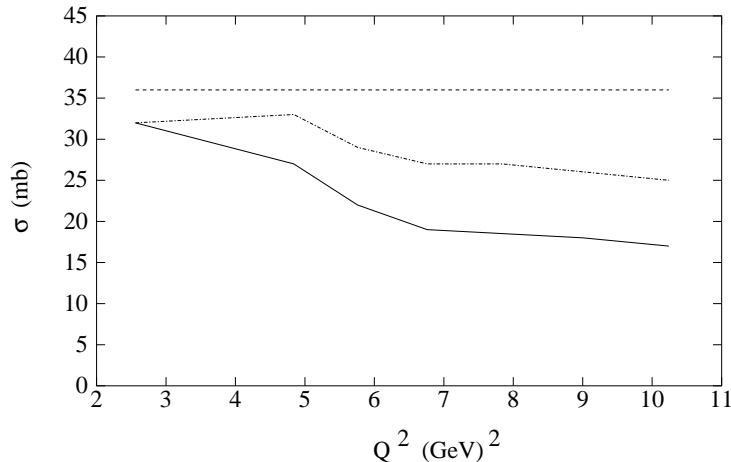
Figure 2: Extracted effective attenuation cross sections $\sigma_{eff}(Q^2)$ as a function of Q^2 exhibit color transparency. The decrease of $\sigma_{eff}(Q^2)$ with Q^2 is sufficiently large that conventional nuclear physics might be ruled out with large Q^2 or sufficient precision. Solid curve: asymptotic wavefunction. Dash-dot curve: KS wave function. The dashed line is the Glauber value of 36 mb.

dependence, but is somewhat ameliorated by studying the nuclear matter limit of large A. Several conclusions are nevertheless well supported by the investigations, reported in more detail elsewhere.[18] These first quantitative perturbative calculations support the claim that exclusive processes are theoretically under better control than the free-space analogues. Next, observables such as the slope of the transparency ratio, sometimes calculated in terms of an ad-hoc "expansion time scale" in hadronic models, probe the interplay of the transverse and longitudinal development of the amplitudes in perturbation theory. The slope depends directly on the x-dependence of wave functions. Next, the calculations show significant reduction in attenuation, even in circumstances where *no rise is seen with Q^2 in transparency ratios*. The explanation, we believe, is that purification to short-distance tends to deplete amplitude normalizations. Division by a free-space cross section, where uncontrolled amplitudes dominate, overestimates the magnitude of hard scattering inside a nucleus, and can give a misleading ratio. This competition is unlikely to be unraveled with Q^2 dependence alone, and calls for another experimetal handle. The A dependence separates the competition between process normalization and attenuation, as revealed by σ_{eff}.

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Acknowledgments

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